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Research Article

Microwave remote sensing of soil moisture, above ground biomass and freeze-thaw dynamic: Modeling and empirical approaches

LAURA ANGELONI, DOMENICO DANIELE BLOISI, PAOLO BURGHIGNOLI, DAVIDE COMITE, DANILO COSTARELLI, MICHELE PICONI*, ANNA RITA SAMBUCINI, ALESSIO TROIANI, AND ALESSANDRO VENERI

ABSTRACT. Human actions have accelerated changes in global temperature, precipitation patterns, and other critical Earth systems. Key markers of these changes can be linked to the dynamic of Essential Climate Variables (ECVs) and related measures, such as Soil Moisture (SM), Above Ground Biomass (AGB), and Freeze-Thaw (FT) Dynamics. ECVs are crucial for understanding global climate changes, including hydrological and carbon cycles. Moreover, monitoring ECVs helps to validate climate models and inform policy decisions. Monitoring activities can be carried out at a global scale by using technologies like microwave remote sensing. However, other than proper technological developments, the study of ECVs requires suitable theoretical retrieval tools, which leads to the solutions of inverse problems. In this survey, we analyze and summarize the main retrieval techniques available in the literature for SM, AGB, and FT, performed on data collected with microwave remote sensing sensors. Furthermore, we present the project *RETINA* (*REmote sensing daTa INversion with multivariate functional modeling for essential climAte variables characterization*), recently funded by the European Union under the Italian National Recovery and Resilience Plan of NextGenerationEU, under the Italian Ministry of University and Research. The main goal of *RETINA*, is to create innovative techniques for analyzing data generated by the interaction of electromagnetic waves with the Earth's surface, applying theoretical insights to address real-world challenges.

Keywords: Microwave remote sensing, essential climate variables, probabilistic cellular automata, neural network operators, Bayesian inversion.

2020 Mathematics Subject Classification: 47A58, 47A63, 47A57, 41A25, 41A05.

1. INTRODUCTION

Human activities such as burning fossil fuels, deforestation, agriculture, and industrial processes are responsible for releasing significant amounts of carbon dioxide (CO_2), methane (CH_4) and other greenhouse gases, driving the rapid modifications that the Earth's climate is experiencing. These actions have caused long-term changes in temperature, precipitation patterns, and other Earth system dynamics.

Key markers of these changes include variations in Essential Climate Variables (ECVs) such as Soil Moisture (SM), Above Ground Biomass (AGB), and Freeze-Thaw (FT) Dynamics. SM dynamics, a crucial part of the global hydrological cycle, are impacted by human-induced climate changes. These dynamics affect water availability, agricultural productivity, and even natural disaster patterns (e.g., droughts and floods). Large-scale deforestation reduces the

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*Corresponding author: Michele Piconi; michele.piconi@unipg.it
https://gcos.wmo.int/en/essential-climate-variables/about

FIGURE 1. The official logo of the *RETINA* project

Earth's capacity to sequester carbon, directly impacting carbon cycles and exacerbating climate change. AGB is a critical measure to monitor carbon stocks and understand the effects of deforestation. Finally, the FT dynamics in polar and boreal regions are influenced by human activity emissions, accelerating permafrost melting and methane release, both of which have significant implications for climate change.

A central role to track these changes is played by Earth observation technologies such as Microwave Remote Sensing (MW RS). The monitoring of variables, like SM, AGB, and FT, helps to validate climate models, understand the feedback mechanisms between human activity and environmental responses, and inform policies aimed at mitigating human impacts on the climate. MW RS utilizes active sensors (e.g., radar) and passive sensors (e.g., radiometers) for continuous monitoring, irrespective of weather or lighting conditions. Depending on the platform (i.e., airborne or spacebone), data collection is possible on regional and global scales [45].

The retrieval techniques in MW RS data are based on theoretical [11], semi-empirical [39], and empirical [46] models, increasingly enhanced by modern machine learning (ML) techniques using neural networks (NN). Theoretical models leverage classical electrodynamics theories, such as, e.g., scattering theory from rough surfaces [45], Radiative Transfer Theory (RTT) [14], Foldy-Distorted Born Approximation (DBA) [40]. Semi-empirical models, such as the Water Cloud Model (WCM) [10], links microwave signals to soil and vegetation parameters. Empirical models, instead, use direct relationships between observed signals and ECVs, and are therefore driven by the physical observable characterized by the microwave sensor. ML is applied to handle complex datasets for more accurate predictions. However, the data sets required for the training are often very large; therefore, significant effort is needed for data annotation. In addition, data and ancillary data are often not continuously available due to acquisition methods, the type of sensor, the spatial and temporal resolution, as well as some practical conditions (e.g., the satellite orbit and the presence of disturbances like clouds).

Very recently, a new research project that aims to propose new methods for the retrieval of the ECVs has been funded, mixing both deterministic and nondeterministic procedures. This endeavour *RETINA* (see Fig. 1, in which the official logo of the *RETINA* project is shown), is funded by the European Union within the framework of the Italian National Recovery and Resilience Plan (NRRP) of the NextGenerationEU program, under the Italian Ministry of University and Research. *RETINA* proposes, for the first time, the application of direct and inverse analytical methods of Approximation Theory based on the theory of the so-called multivariate neural network (NN) operators (see, e.g., [16, 18, 19]) for the modeling and estimation of

SM, AGB, and FT, using data from space missions. In the context of analytical methods in Approximation Theory, we can mention the following significant contributions [1]-[4].

The fundamental idea in RETINA is to combine analytic inversion techniques (based on functional analysis tools, such as series expansions [33]) and Bayesian approaches performed in conjunction with Monte Carlo methods. The main two (complementary) strategies of *RETINA* can be summarized as follows:

(1) Data Modeling with well-known Multivariate NN Operators: Through functional analysis techniques, theoretical inversion of these operators is achieved, resulting in an approximate analytical model for the target geophysical variables that is useful for their estimation. To address potential data disturbances, as well as take into account the uncertainty of the model, the NN operators will be extended to have the possibility of representing interval-valued fuzzy sets (IVFS), which allow for the representation of (uncertain variables) situations that are more coherent with real-world situations.

(2) Bayesian Inversion with Monte Carlo Markov Chain (MCMC): These methods are used to sample from the posterior distribution, a robust technique for Bayesian inversion. Through MCMC techniques, Bayesian inversion complemented the NN operator approach. *RETINA* targets the introduction of a specific type of Markov Chain, Probabilistic Cellular Automata (PCA), characterized by a parallel updating rule, which is expected to be particularly effective for retrieving multi-component physical quantities, such as matrix-formatted data.

To set the state of the art of the retrieval techniques, a part of the *RETINA* project explores and summarizes the main algorithms available in the literature, focusing on the retrieval methods applied to the characterization of the bio-geophysical variables of interest. Providing an overview of these algorithms is the main motivation for this paper.

The remainder of this paper is organized as follows. Section 2 present a description of existing approaches for monitoring bio-geophysical variables by using Microwave Remote Sensing. Section 3 presents a freely accessible dataset of remote sensing data that can be used for training, testing, and benchmarking retrieval procedures for ECVs. Sections 4, 5, 6 describe recent techniques for the retrieval of vegetation biomass, freeze-thaw, and soil moisture, respectively. Section 7 discusses potential future developments of the theory. Finally, conclusions are drawn in Section 8.

2. Microwave Remote Sensing for Monitoring Bio-geophysical Variables

MW RS is a powerful tool for monitoring bio-geophysical variables such as SM, AGB, and FT. This is given by capability to operate under all weather conditions, during day and night, and by the possibility of penetrating clouds, rain, and vegetation. Although often characterized by low spatial resolution (i.e., with respect to optical sensors), MW RS can generally offer relatively high temporal resolution with revisit time in the order of days, and up to hours in certain specific conditions (e.g., in the presence of satellite constellations).

Microwave signals are sensitive to the dielectric properties of soil, water, and vegetation, and are capable of penetrating surface layers (in the order of centimeters, depending on the wavelength and on the structure of the media), making them suitable for observing both superficial and shallow phenomena.

Active sensors, including Synthetic Aperture Radar (SAR) systems, emit microwave radiation and collect the reflected or scattered signals. Passive sensors, such as radiometers, measure naturally emitted thermal radiation. MW radiation wavelengths range from 1 mm to 1 meter and are divided into different frequency bands (e.g., L-band, C-band, X-band and P-band),

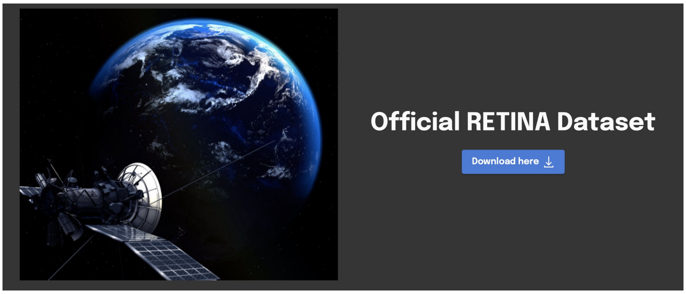

FIGURE 2. A screenshot of the *RETINA* dataset web page, available at the following link: `https://retina.sites.dmi.unipg.it/dataset.html`

which correspond to decreasing wavelength. In MW RS, L-band (15–30 cm wavelength) and C-band (4–8 cm wavelength) are commonly used thanks to their balance between penetration depth, spatial resolution, and the technological maturity of the system. Longer wavelengths (such as L-band or P-band, the latter planned to be used in the future ESA's Biomass mission [50]) are preferred for dense vegetation and high-biomass areas, even though they may suffer signal saturation. On the other hand, shorter wavelengths (e.g., C-band) are effective for less dense biomass or canopy surface and agricultural observations. Missions like SMAP [25] and SMOS [35] use both active and passive microwave sensors to monitor SM globally. Moreover, microwave sensors like AMSR-E [49] and CryoSat [60] were pivotal in polar studies and provided important data to help tracking sea surface temperatures, sea ice extent, and thickness.

Even though MWRS is a fundamental tool for measuring and monitoring many ECVs, performing these tasks is not straightforward. Among others, the following challenges have to be addressed: signal saturation that occurs in dense forests or very high biomass regions where backscatter no longer increases with increasing biomass; spatial resolution that can be improved by combining microwave data with optical, thermal, and in-situ observations for a more comprehensive understanding of Earth's systems; difficulty in separating contributions from soil, vegetation, and atmospheric layers in mixed environments. To overcome these limitations, satellite missions like ESA's Biomass [50], NISAR [22], and HydroGNSS [58] aim to improve data resolution, and extend observational capabilities. Moreover, alongside model-based approaches, AI techniques are increasingly used to analyze complex microwave datasets to obtain more accurate predictions and insights.

3. RETINA Dataset of Microwave Remote Sensing Data

One of the main goals of the *RETINA* project is to release a freely accessible dataset of remote sensing data that can be used for training, testing, and benchmarking retrieval procedures for the considered ECVs. All the selected images have been collected in a dedicated open-access repository available from the *RETINA* website (see Fig. 2) at the following link: `https://retina.sites.dmi.unipg.it/dataset.html`.

The source of all the images (which can be downloaded for free, as stated in the copyright section of this paper) is the Sentinel-1 ([43, 56]) satellite constellation, the first of the ESA's Copernicus Program. The images are available in GeoTIFF format with VV and VH bands. To handle these images, it is possible to use any GIS software, such as QGIS or SNAP. Examples of RS images that can be found in the *RETINA* dataset are shown in Fig. 3.

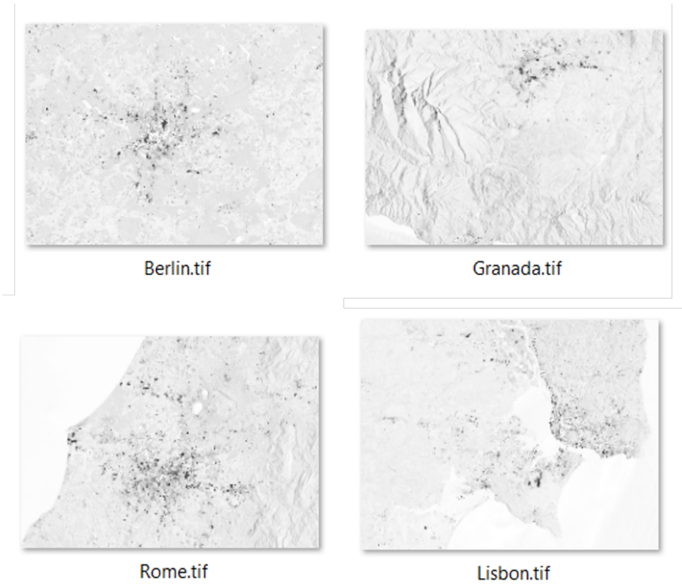

FIGURE 3. Four RS images extracted from the *RETINA* dataset. On the top (from left to right): Berlin (Germany) and Granada (Spain) areas. On the bottom: (from left to right): Rome (Italy) and Lisbon (Portugal) areas

To help users of the dataset, a Python script for reading the images is available on the *RETINA* website. The script allows the user to clip the pixel intensities between user-chosen minimum and maximum threshold values to customize how images are displayed.

Fig. 4 shows the functional scheme of the Python script for reading the .tif files in the Official RETINA Dataset. The input file is processed to produce two new images, one for Band 1 (clipped and normalized) and one for Band 2 (clipped and normalized). In addition to the Python script, also a MATLAB script is available on the *RETINA* website.

Below, we provide a short review of the main algorithms available in the literature for each one of the target ECVs considered in *RETINA*.

4. Vegetation Biomass Retrieval: an Overwiew

Retrieving vegetation biomass, specifically AGB, is crucial for understanding carbon cycles, forest dynamics, and the role of vegetation in climate change mitigation. MW RS, particularly radar systems, has emerged as a key method for estimating AGB due to its ability to penetrate vegetation layers and provide detailed structural information [30].

Longer Wavelengths (L-band, P-band) penetrate deeper into vegetation, interacting with trunks and larger branches, making them ideal for high-biomass regions. Shorter Wavelengths (e.g.,C-band) are sensitive to canopy features like leaves and smaller branches.

Techniques for Biomass Retrieval include:

- Theoretical approaches, such as the Radiative Transfer Theory (RTT) and the Wave Theory. RTT focuses on the principles of energy conservation and the transfer of energy through disordered media (i.e., layers of vegetation, see, e.g., [15, 38]). A notable example is the Michigan Microwave Canopy Scattering model (MIMICS) [57], which

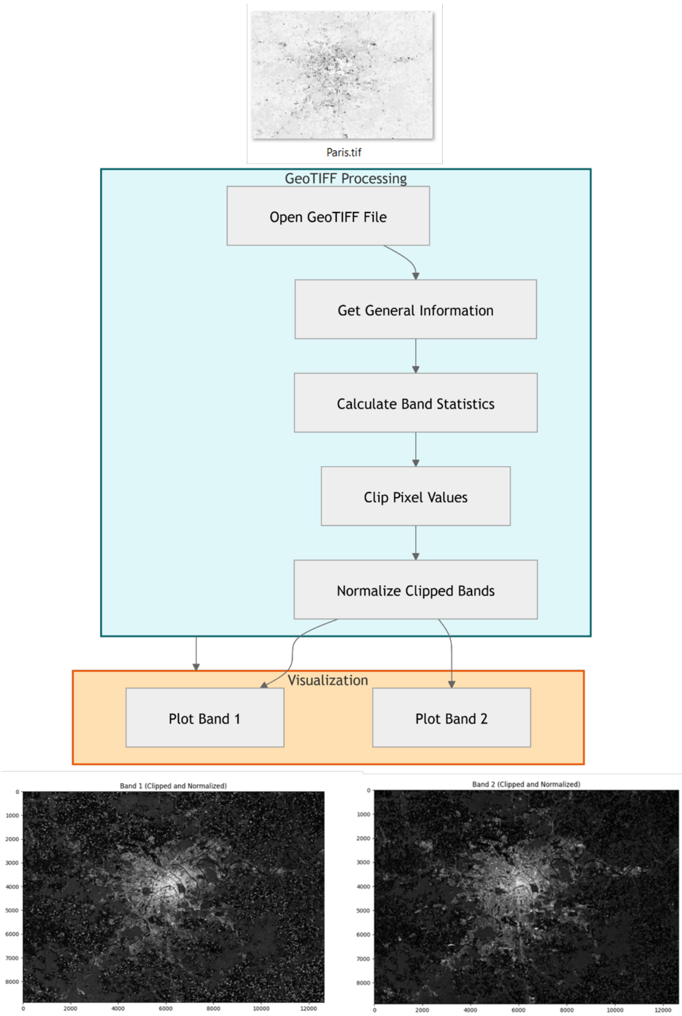

FIGURE 4. The functional scheme of the Python script for reading the Official RETINA Dataset

divides the vegetation into crown, trunk, and ground components, and solves the vector radiative transfer equation (VRTE) iteratively. The Wave Theory, on the other hand, approximates solutions to Maxwell's equations to describe scattered electromagnetic fields. An example of this class is the Distorted Born Approximation (DBA), which uses electromagnetic wave theories of scattering to simulate how microwaves interact with vegetation structures. DBA represents the vegetation layer as a random collection of individual scatterers (see, e.g., [41]). These models are highly accurate but computationally intensive, also requiring a large collection of input data.

- Semi-empirical, such as the Water Cloud Model (WCM), models are simplified models that relate the measured radar backscattering coefficient to biomass using empirical relationships. WCM combines contributions from vegetation and ground backscatter (see, e.g., [42, 47]). These methods are adjusted for vegetation density and structure

using parameters like canopy height and attenuation and are often used with regression techniques to estimate AGB from radar data. As the backscattering coefficient easily saturates at very high biomass densities, limiting the retrieval of AGB beyond a certain threshold, interferometric techniques are also often adopted as coherence appears to be effective to identify forested/nonforested areas and the height of the canopy [8]. The main observable is the complex degree of coherence (CDC), determining the potential of two electromagnetic signals to interfere [12], which can be extracted from the data and compared with models (based on the WCM) accounting for stem volumes, tree height, and fill-factor, i.e., the fraction of ground covered by trees [53].
- ML techniques utilize algorithms like NN and support vector machines to model complex relationships between radar signals and biomass (see [6] for a complete review on this topic). They can integrate multi-source data (e.g., optical and microwave) to improve AGB retrieval accuracy.

SM, surface roughness, and vegetation water content can complicate signal interpretation. In general, retrieval models must account for different vegetation types, structures, and climates to ensure accuracy.

Current and upcoming radar missions for vegetation biomass studies are: ESA Biomass Mission (P-band SAR), designed specifically for global forest AGB mapping; NISAR (L- and S-band SAR), which targets forest structure and biomass dynamics; Sentinel-1 (C-band SAR) that provides data for biomass monitoring with limited penetration depth; GNSS Reflectometry, that is a more recent technique using reflected GPS signals (e.g., CYGNSS, TDS-1) to assess the AGB distribution at global scale ([51]).

5. Freeze-Thaw Retrieval: An Overview

FT retrieval focuses on monitoring seasonal transitions in the soil's thermal state, particularly between frozen and unfrozen conditions. These transitions are critical for understanding the water cycle, energy balance, and greenhouse gas dynamics, especially in high-latitude regions where permafrost melting can release significant amounts of methane [29, 48].

MW RS has proven particularly well-suited for FT detection due to its high sensitivity to phase changes of water in soil, as the permittivity of water decreases dramatically between liquid and solid states [45]. Both active and passive MW remote sensing techniques are currently employed in the field, addressing the problem from different perspectives. SAR and radiometer systems (e.g., Sentinel 1 and SMAP) provide high-resolution spatial data by capturing the dynamic temporal variations of the corresponding physical observable, which are associated with seasonal changes in FT states [23, 37]. However, these systems are constrained by limited temporal resolution, typically on the order of days [13]. Passive MW RS instruments, like SMAP, measure naturally emitted microwave radiation, wherein the measured power is expressed in terms of blackbody equivalent radiometric temperature (or radiometer brightness temperature). Radiometers provide FT-related data with high temporal resolution compared to radar systems, $\sim$ 3 days, at the expense of a low spatial resolution [23]. This limitation arises from the relatively weak electromagnetic signal emitted by the natural media, thus requiring a vast observation region to detect meaningful signals. Very recently, GNSS Reflectometry (GNSS-R) has been successfully adopted in the field, addressing the limitations of traditional active and passive measurement techniques and providing sensible data with both high temporal and spatial resolution ([52]). This method involves detecting signals transmitted by a constellation of GNSS satellites and scattered off Earth's surface. Since it exploits signals originally designed for GPS purposes, it is often referred to as a signal of opportunity technique. In this framework, the measured observable is the surface reflectivity for left-hand circularly polarized fields, related to the dielectric function, in turn connected to climate variables, through the Fresnel's coefficients.

FT retrieval methods include:

- Theoretical models, mostly developed for passive radiometric measurements. This approach aims to estimate the brightness temperature from the physical properties of the medium by means of radiative transport theories. A notable example is the Helsinki University of Technology (HUT) model [44], which solves the scalar radiative transfer equation for multilayered systems.
- Empirical approaches, wherein thresholding is a commonly adopted method. For active measurements, the collected backscattering coefficient is compared with its reference values for the thaw and freeze state, with the goal of determining the present water phase [23]. For GNSS systems the same method is adopted wherein the seasonal threshold algorithm now involves reference reflectivity values and measurements. Radiometric systems also employ a thresholding technique, but in this case, freeze/thaw discrimination is based on the difference between the vertical and horizontal polarizations of the brightness temperature.
- ML algorithms like Random Forest or NN process large datasets to detect patterns in FT transitions. These techniques are useful for integrating multi-sensor and auxiliary data (e.g., temperature and vegetation cover).

Problematics related to FT retrieval can include: vegetation and snow layers can obscure FT signals; rapid transitions may be missed without frequent observations; factors like soil roughness, composition, and moisture can complicate FT retrieval accuracy. New systems like ESA Biomass will enhance FT monitoring capabilities.

6. Soil Moisture Retrieval: An Overview

SM is a key bio-geophysical variable influencing global water cycles via evapotranspiration processes, exchange of heat between land and near-surface atmosphere, energy balance, and biochemical/carbon cycles [55]. Recently, MW RS has emerged as a valuable tool for real-time SM monitoring due to its sensitivity to the soil-water ratio, which alters both the medium's emissivity and the backscattering properties of signals, opening the possibility for active and passive RS measurements. Key missions and instruments for SM study are: SMAP that combines SAR and radiometer data for global soil moisture monitoring; SMOS that uses passive L-band radiometer for large-scale SM and salinity observations; Sentinel-1 C-band SAR that provides high-resolution SM data, especially useful for agricultural applications.

Techniques for SM retrieval include:

- Theoretical models are particularly relevant in radar active measurements. Among them, the Integral Equation Method (IEM), based on electromagnetic scattering theory, has gained prominence over time and is now the most widely used [26]. As active measurements are particularly affected by surface roughness conditions, the theory statistically accounts for the random variations of scattering surfaces, estimating the average values of scattered power as a function of surface roughness and the soil dielectric function.
- Semi-empirical models, which are widely preferred due to their effectiveness and relatively simple implementation. For active systems, we distinguish two types of semi-empirical approaches. The first is based on the experimental calibration of the IEM,

which corrects deviation of the theory from measurements correcting roughness effects [9]. The second exploits knowledge of scattering behavior in specific limiting cases, combined with experimental observations, to create ready-to-use formulas derived through data fitting. The most used frameworks are the Oh and the Dubois models [24, 54], both directly relating radar backscattering with volimetric soil moisture and roughness. For passive measurements, semi-empirical models are based on the scalar radiative transfer equation, solved at zeroth order and adjusted using experimental parameters to account for surface roughness, the mixing of different polarization components, and the influence of vegetation and atmospheric layers [36, 59]. Semi-empirical models have also been used for GNSS-based SM retrieval, revealing the potential of this novel technology to provide excellent results even at global scale [7].
- ML and Data Assimilation: ML algorithms analyze complex, multi-sensor datasets to improve accuracy; data assimilation integrates satellite observations with hydrological models for comprehensive SM monitoring. For instance, Convolutional Neural Networks (CNNs) are useful for processing SAR data [32], as they can extract spatial features from radar images, providing enhanced accuracy. More complex NN architecture can describe spatial features along with also temporal evolution.

Dense vegetation and uneven surfaces can obscure SM signals, so ground-truth data are often requested to ensure accuracy in various terrains and climates.

7. Future Developments: Neural Network Operators and Bayesian Inversion

NNs have become highly popular due to their utility across numerous fields, including Artificial Intelligence (AI), ML, and Approximation Theory (ATh). Within the *RETINA* project, NNs will be applied in the context of ATh to develop approximate analytical models and their inversions for specific ECVs. In relation to the theory of NNs, in [19], the authors explored the functional properties of the NN operators. This research highlights the potential of these operators in modeling general two-dimensional structures, such as SAR satellite images ([27]) or, more in general, RS data.

For the sake of completeness, we recall the definition of such operators in both their classical and Kantorovich form. The multivariate discrete NN operators can be defined as follows:

$$F_n^d(f,\underline{x}) \;:=\; \frac{\displaystyle\sum_{\underline{k}\in\mathcal{J}_n} f\left(\frac{\underline{k}}{n}\right)\Psi_\sigma(n\underline{x}-\underline{k})}{\displaystyle\sum_{\underline{j}\in\mathcal{J}_n}\Psi_\sigma(n\underline{x}-\underline{k})}, \quad \underline{x}\in Q^d := [a_1,b_1]\times\cdots\times[a_d,b_d], \tag{7.1}$$

where the function

$$\Psi_\sigma(\underline{x}) := \phi_\sigma(x_1)\cdot\phi_\sigma(x_2)\cdots\phi_\sigma(x_d), \qquad \underline{x} := (x_1,...,x_d)\in\mathbb{R}^s \tag{7.2}$$

is the multivariate (tensor-product) density function defined by means of suitable sigmoidal function $\sigma:\mathbb{R}\to\mathbb{R}$ ([20]), the set of indexes

$$\mathcal{J}_n := \left\{\underline{k}\in\mathbb{Z}^d :\ \lfloor na_i\rfloor \le k_i \le \lceil nb_i\rceil\right\}$$

and

$$\phi_\sigma(x) \;:=\; \frac{1}{2}\left[\sigma(x+1)-\sigma(x-1)\right], \quad x\in\mathbb{R}.$$

We recall that, by Cybenko's definition in [20], $\sigma:\mathbb{R}\to\mathbb{R}$ is called a *sigmoidal function* if $\lim_{x\to-\infty}\sigma(x)=0$ and $\lim_{x\to+\infty}\sigma(x)=1$.

While, the Kantorovich NN operators are:

$$K_n^d(f, \underline{x}) \ := \ \frac{\displaystyle\sum_{\underline{k} \in \mathcal{J}_n} \left[n^d \int_{R_{\underline{k}}^n} f\left(\underline{u}\right)\, d\underline{u} \right] \Psi_\sigma(n\underline{x} - \underline{k})}{\displaystyle\sum_{\underline{j} \in \mathcal{J}_n} \Psi_\sigma(n\underline{x} - \underline{k})}, \quad \underline{x} \in Q^d, \tag{7.3}$$

where

$$R_{\underline{k}}^n := \left[\frac{k_1}{n}, \frac{k_1+1}{n}\right] \times \cdots \times \left[\frac{k_d}{n}, \frac{k_d+1}{n}\right] \tag{7.4}$$

are suitable multidimensional rectangles in which we will compute certain mean values of the considered function $f : Q^d \to \mathbb{R}$.

We stress that, with respect to the classical (non-deterministic) theory of shallow and deep NNs, the NN operators are instead widely studied (see. e.g., [16, 17]) mathematical operators that are suitable to pursue a deterministic modeling approach, and also an enhancement/rescaling one.

The task of inverting the above NN operators will be based on an analytical strategy, such as the possibility of exploiting Laurent's series, particularly when working with operators in Hilbert spaces, or methods of Approximation Theory. To address 2D data affected by measurement errors or other disturbances, interval-valued fuzzy sets (IVFS) have been proposed as a robust modeling tool (see [5]).

The NN operators approach will be enhanced through the integration of Bayesian inversion, which leverages advanced Monte Carlo Markov Chain (MCMC) techniques. These new techniques utilize a parallelized transition kernel to enable efficient sampling from the posterior distribution. In the Bayesian framework, the variable to retrieve is treated as a random variable. With this approach, the retrieval procedure aims to determine the probability distribution of this variable given the observed data (posterior probability distribution). Then, the outcome of the retrieval procedure is the value that maximizes the posterior probability distribution. A standard way to estimate such probability distribution is to simulate the evolution of a Markov chain designed so that its stationary distribution is the posterior probability distribution of the variable to retrieve. Since the empirical distribution of the Markov chain converges to its stationary distribution, if the chain is run for a sufficiently long time, its empirical distribution is an estimate of the posterior probability distribution of the variables to retrieve. In this context, algorithms commonly used to simulate the Markov chain include the Metropolis algorithm, the Metropolis-Hastings algorithm, and the Gibbs sampler. If the Markov chain has multi-component states, such as in the case of 2D data, the previous algorithms sample the next state of the chain from a set of neighbors of the current state differing in only one component. This strategy is referred to as single-flip dynamics [31]. An alternative to single-flip dynamics is the use of Probabilistic Cellular Automata (PCA) [28]. In PCA, all components of the state are updated simultaneously and independently at each step. This approach expands the set of neighbors to include the entire state space, resulting in higher motility and potentially faster convergence to equilibrium.

More formally, a PCA, is a Markov chain $(X_n)_{n\in\mathbb{N}}$ defined on $\mathcal{X} = \{1, \ldots, k\}^N$, where N is the number of components of the system, and whose transition matrix is such that

$$\mathbb{P}\{X_n = \tau | X_{n-1} = \sigma\} = \prod_{i=1}^{N} \mathbb{P}\{(X_n)_i = \tau_i | X_{n-1} = \sigma\}. \tag{7.5}$$

A transition matrix of this type is obtained by defining a function $H : \mathcal{X} \times \mathcal{X} \to \mathbb{R}$ of type $H(\sigma,\tau) = -\sum_{1\leq i\leq N} h_i(\sigma)\tau_i$ and transition probabilities as

$$\mathrm{P}(\sigma,\tau) = \frac{e^{-\beta H(\sigma,\tau)}}{Z_\sigma} = \prod_{1\leq i\leq N} \frac{e^{\beta h_i(\sigma)\tau_i}}{(Z_\sigma)_i} \tag{7.6}$$

where β is a positive parameter called the inverse temperature and Z_σ is a normalizing constant whose knowledge is not known to simulate the evolution of the chain. Then, if $h(\cdot)$ satisfies certain suitable conditions, the stationary measure of the chain can be proven to be $\pi(\sigma) = \frac{\sum_\tau \mathrm{P}(\sigma,\tau)}{\sum_{\sigma,\tau} \mathrm{P}(\sigma,\tau)}$ [21, 34]. The knowledge of the stationary measure of the chain allows to tune the algorithm so to favor the sampling of the more useful configurations [21, 34].

PCA's inherent parallelism offers significant computational advantages, particularly when leveraging massively parallel processing hardware such as GPUs or TPUs. These processors enable simultaneous updates of all components at each step, greatly enhancing efficiency.

8. Conclusions

Climate change is one of the biggest challenges Mankind is called to face. The precise estimation and monitoring of the ECVs on a global scale is a fundamental step to describing and understanding the rapid changes the Earth's climate is experiencing and, consequently, to determine the more appropriate actions to mitigate the adverse effects of these changes.

Microwave remote sensing is a cornerstone of modern Earth observation, enabling critical insights into climate dynamics, environmental monitoring, and resource management at local, regional, and global scales. This paper highlights advancements in microwave sensing technologies and their integration with ML to enhance the monitoring of Earth's bio-geophysical processes.

Particular emphasis is given to those retrieval techniques that will be exploited to estimate the target ECVs in the context *RETINA* project.

Data Availability Statement

The data made available in the framework of the *RETINA* project can be obtained from the "RETINA dataset" at `https://RETINA.sites.dmi.unipg.it/dataset.html`. *RETINA* Code and documentation are available through the website `https://RETINA.sites.dmi.unipg.it`. Permission is granted to use, copy, or modify the documentation for educational and research purposes without fee.

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Laura Angeloni
University of Perugia
Department of Mathematics and Computer Science
Via Vanvitelli, 1, Perugia, Italy
Email address: laura.angeloni@unipg.it

Domenico Daniele Bloisi
International University of Rome UNINT
Department of International Humanities ans Social Sciences
Via Cristoforo Colombo, 200, Rome, Italy
Email address: domenico.bloisi@unint.eu

Paolo Burghignoli
Sapienza University of Rome
Department of Information Engineering, Electronic and Communications
Via Eudossiana, 18, 00184, Roma, Italy
Email address: paolo.burghignoli@uniroma1.it

Davide Comite
Sapienza University of Rome
Department of Information Engineering, Electronic and Communications
Via Eudossiana, 18, 00184, Roma, Italy
Email address: davide.comite@uniroma1.it

Danilo Costarelli
University of Perugia
Department of Mathematics and Computer Science
Via Vanvitelli, 1, Perugia, Italy
Email address: danilo.costarelli@unipg.it

Michele Piconi
University of Perugia
Department of Mathematics and Computer Science
Via Vanvitelli, 1, Perugia, Italy
Email address: michele.piconi@unipg.it

Anna Rita Sambucini
University of Perugia
Department of Mathematics and Computer Science
Via Vanvitelli, 1, Perugia, Italy
Email address: anna.sambucini@unipg.it

ALESSIO TROIANI
UNIVERSITY OF PERUGIA
DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE
VIA VANVITELLI, 1, PERUGIA, ITALY
Email address: alessio.troiani@unipg.it

ALESSANDRO VENERI
SAPIENZA UNIVERSITY OF ROME
DEPARTMENT OF INFORMATION ENGINEERING, ELECTRONIC AND COMMUNICATIONS
VIA EUDOSSIANA, 18, 00184, ROMA, ITALY
Email address: alessandro.veneri@uniroma1.it